University of Saskatchewan
Department of Physics and Engineering Physics
EP 228.3
Final Examination
A CLOSED BOOK EXAMINATION

[Examiner D.A. Degenstein]

Time: 3 hours. **Date: April, 2005**

Instructions: Candidates are to answer ***ALL*** questions in the booklets provided.
There are 13 questions in the exam.
All questions ***DO NOT*** have the same value.
Electronic calculators are required.
One formula sheet is allowed.
Please read each question carefully before attempting it.
THINK before you act!!!!

1) Express each of the complex expressions in terms of $z = x + iy$. Show some work or you get no marks. ***(12 marks)***

i) $z = \dfrac{dz_1^2}{dt}$ where $z_1 = 5 - i2t$

ii) $z = \dfrac{(z_1^* - z_2)}{z_2}$ where $z_1 = 2e^{i\frac{4\pi}{3}}$ and $z_2 = 2e^{-i\frac{\pi}{3}}$

iii) $z(3,0)$ where $z(x,t) = \dfrac{d^2 e^{z_1}}{dx^2}$ and $z_1 = -i\dfrac{\pi}{3}t - 2x$

iv) $z = z_1 z_2$ where $z_1 = (8 + i12)^{-1}$ and $z_2 = 2 - i3$

2) Show, using the complex representation of $\cos\theta$ and $\sin\theta$, that: ***(5 marks)***

$$[\tan(\theta_1) - \tan(\theta_2)]\cos(\theta_1)\cos(\theta_2) = \sin(\theta_1 - \theta_2)$$

3) What are the non-zero frequency components, a_n and b_n values, and the angular frequencies of these components for the function

$$f(t) = 6[\tan(60\pi t) - \tan(20\pi t)]\cos(60\pi t)\cos(20\pi t) - 5\cos(40\pi t) + 1.2\sin(40\pi t) - 3.8$$

on the interval $[-0.1, 0.1]$? ***(5 marks)***

4) Assume the number of Uranium molecules in an experiment, N_t, can be defined at any time as $N_t = N_{t-1}r - A$. In this expression A and r are constants and N_{t-1} is the number of Uranium molecules at the previous time step. Prove by induction that the number of Uranium molecules in the system at any time t is given by

$$N_t = N_0 r^t - A\frac{(1-r^t)}{(1-r)}$$

where N_0 is the number of molecules at time $t = 0$ *s*. ***(5 marks)***

5) What are the Fourier coefficients, a_n and b_n, for the function shown in Figure 5.1? Assume T_0 = -3.0 *s* and T_f = 3.0 *s*. ***(8 marks)***

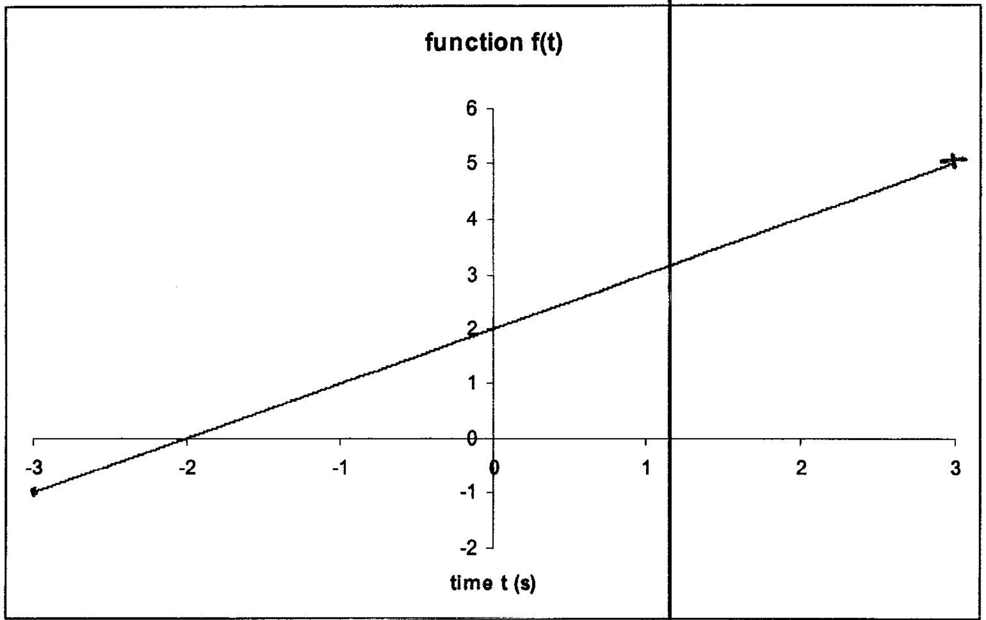

Figure 5.1: Estimate the required points by reading them from the graph.

6) Given the matrix $A = \omega\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and the definition $e^{At} = I + \frac{At}{1!} + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \frac{A^4t^4}{4!} + \cdots$ show that $e^{At} = \begin{pmatrix} \cos(\omega t) & -\sin(\omega t) \\ \sin(\omega t) & \cos(\omega t) \end{pmatrix}$.

(6 marks)

7) At a height h above the surface of the Earth the gravitational force on a mass m is given by

$$F = \frac{mMG}{(R+h)^2} \tag{7.1}$$

where M and R are respectively the mass and radius of the Earth and G is Newton's gravitational constant. For small values of h it is common to use the approximation $F = mg$ where $g = \frac{MG}{R^2}$. If more precision is required for a particular calculation what are the next two correction terms in the Taylor series expansion of equation 7.1. ***(5 marks)***

8) Given the matrix $A = \begin{pmatrix} -3 & -1 \\ 1 & 0 \end{pmatrix}$: ***(10 marks)***

i) What is the determinant of A?

ii) What is the inverse of A?

iii) What is x in $Ax = \begin{pmatrix} 2 \\ i \end{pmatrix}$?

iv) What are the eigenvalues of A?

v) What are the eigenvectors of A?

9) Assume $p = (-5,2)$ is a point in a two-dimensional Cartesian coordinate system represented by the two unit vectors $(\hat{x}, \hat{y})$ then: ***(9 marks)***

i) Given the vectors $\vec{v}_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ what are the constants that multiply $\vec{v}_1$ and $\vec{v}_2$ such that the point p is a linear combination of $\vec{v}_1$ and $\vec{v}_2$?

ii) Given the vectors $\vec{v}_1 = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{pmatrix}$ what are the constants that multiply $\vec{v}_1$ and $\vec{v}_2$ such that the point p is a linear combination of $\vec{v}_1$ and $\vec{v}_2$?

iii) The vectors $\vec{v}_1$ and $\vec{v}_2$ in part ii) represent a coordinate system that is rotated by a particular angle with respect to the original $(\hat{x}, \hat{y})$ coordinate system. What is the value of the vector p, in the $(\hat{x}, \hat{y})$ coordinate system, if it is rotated by the same amount as $\vec{v}_1$ and $\vec{v}_2$?

$X^3 + 3x^2(-3) + 3x(9) + (-3$

10) Take a Taylor Series expansion of $f(x) = -2 + 3(x-3) - 8(x-3)^2 + 2.5(x-3)^3$ about the point $x = 3$. What are the first three non-zero coefficients of this expansion? What is the value of the MacLaurin series expansion of this function evaluated at $x = 4$? ***(5 marks)***

11) Given the following data set : ***(9 marks)***

X	f(x)
-10	-26
-5	14
0	4
5	-56
10	-166

Table 11.1

i) Find the coefficients of the quadratic $f(x) = a_2x^2 + a_1x + a_0$ that goes through the two end-points and the middle point of this data set.

ii) What are the values of this quadratic at x = *-5* and x = *5*?

iii) What are the best approximations for the numerical derivatives at each of the relevant data points?

12) If z_1 and z_2 are two complex numbers, as seen in Figure 12.1 below, show that $z_1 z_2^* + z_1^* z_2 = 2|z_1||z_2|\cos\theta$. Also, use this result and Figure 12.2 to prove, $|z_3|^2 = |z_1|^2 + |z_2|^2 - 2|z_1||z_2|\cos\theta$, the Law of Cosines. ***(6 marks)***

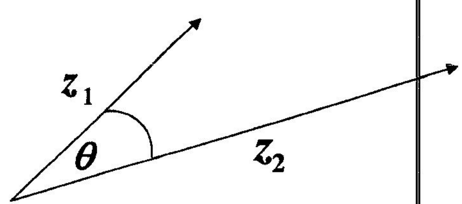

Figure 12.1

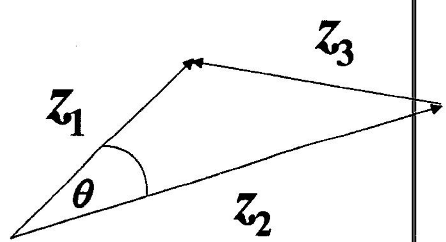

Figure 12.2

13) Given the following data :

x	$f(x)$	$f'(x)$
2	9	
3		6
4		1
5		-7
6		-1
7	13	

where the derivative was taken using the central difference method what is the original data set? ***(5 marks)***

- End of Examination -